

k-MOND (WITH "DARK MATTER" AS A DISTINCTION BETWEEN INERTIAL AND GRAVITATIONAL MASS)

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Abstract

In this work we definitely prove a possibility that Milgrom's modified Newtonian dynamics, MOND, can be consistently interpreted as a theory with the modified kinetic terms of the usual Newtonian dynamics, simply called k-MOND. Precisely, we suggest only a functional dependence between inertial and gravitational mass tending toward identity in the limit of large accelerations (characteristic for Newtonian dynamics and its relativistic generalizations) but which behaves as a principal non-identity in the limit of small accelerations (smaller than Milgrom's acceleration constant). This functional dependence implies a generalization of the kinetic terms (without any change of the gravitational potential energy terms) in the usual Newtonian dynamics including generalization of corresponding Lagrange formalism. Such generalized dynamics, k-MOND, is identical to Milgrom's MOND. Also, mentioned k-MOND distinction between inertial and gravitational mass would be formally treated as "dark matter".

In this work we shall definitely prove a possibility that Milgrom's modified Newtonian dynamics, MOND [1]-[11], can be consistently interpreted as a theory with the modified kinetic terms of the usual Newtonian dynamics, simply called k-MOND. Precisely, we shall suggest only a functional dependence between inertial and gravitational mass tending toward identity in the limit of large accelerations (characteristic for Newtonian dynamics and its relativistic generalizations) but which behaves as a principal non-identity in the limit of small accelerations (smaller than Milgrom's acceleration constant). This functional dependence implies a generalization of the kinetic terms (without any change of the gravitational potential energy terms) in the usual Newtonian dynamics including generalization of corresponding Lagrange formalism. Such generalized dynamics, k-MOND, is identical to Milgrom's MOND. Also, mentioned k-MOND distinction between inertial and gravitational mass would be formally treated as "dark matter". (Name k-MOND is directly inspired by name k-essence [12]-[14] since in

both cases it is supposed that change of the usual dynamics is realized by change of the usual kinetic terms by introduction of the more complex kinetic terms. However any closer correlation between k-essence theories and k-MOND is not supposed apriorily.)

As it is well-known [1]-[11], for a satisfactory theoretical explanation of the observationally obtained flat galaxy rotation curves, Tully-Fisher law etc., Milgrom suggested a modification of the Newtonian classical dynamics, called MOND.

In MOND it is supposed that there is a new natural constant representing Milgrom's acceleration $a_M \simeq 10^{-10} \frac{m}{s^2}$. It is supposed too that for system total acceleration vector \mathbf{a} with extremely small absolute value a , comparable with Milgrom acceleration, Newtonian classical dynamical law of the physical system must be changed in the practically the following form

$$F \simeq ma \frac{a}{a_M}. \quad (1)$$

Here m represents the system mass and a - the system acceleration absolute value. For example, for a galaxy with mass M , that acts by Newtonian gravitational force $F = G \frac{mM}{R^2}$ at a periferically rotating star with mass m , speed v and small centrifugal acceleration absolute value $a = \frac{v^2}{R}$ at distance R in respect to galaxy center, where G represents the Newtonian gravitational constant, MOND dynamics of this star (1) implies

$$G \frac{mM}{R^2} \simeq m \left(\frac{v^2}{R} \right)^2 \frac{1}{a_M} \quad (2)$$

and further

$$v^4 \simeq a_M \quad (3)$$

corresponding excellently to Tully-Fisher relation for spiral galaxies and many other relevant astronomical observational data.

On the other hand it is supposed within MOND that for usual, large acceleration absolute value, much larger than Milgrom's acceleration, classical dynamical law must have approximately usual Newtonian form

$$F \simeq ma. \quad (4)$$

In general case, i.e. for arbitrary small or large acceleration absolute value, MOND supposes the following dynamical law

$$F = ma \mu \left(\frac{a}{a_M} \right) \quad (5)$$

where $\mu \left(\frac{a}{a_M} \right)$ represents a modification function depending of a as the variable and a_M as the parameter so that for small acceleration absolute value (5) tends asymptotically toward (1) while for large acceleration absolute value (5) tends asymptotically toward (4).

Even if MOND simply and elegantly describes important astronomical observational data physical interpretation or nature of the MOND is not simple at all. MOND can be an introduction in a quite new physics or it can be a simple calculation algorithm, i.e. appropriate approximation formalism. Now we shall suggest the following physical interpretation of the MOND.

Firstly, we shall suggest the following functional dependence between inertial mass, m_i , and gravitational mass, m_g , tending toward identity in the limit of large accelerations (characteristic

for Newtonian dynamics and its relativistic generalizations) and behaving as a principal non-identity in the limit of small accelerations (smaller than Milgrom's acceleration constant) in the following way

$$m_i = m_g \left(\frac{a^2}{a^2 + \frac{a_M^2}{2^2}} \right)^{\frac{1}{2}} = m_g \left(\frac{a^2}{a^2 + \frac{a_M^2}{2^2}} \right)^{\frac{1}{2}} = m_g f(a^2) \quad (6)$$

where

$$f(a^2) = \left(\frac{a^2}{a^2 + \frac{a_M^2}{2^2}} \right)^{\frac{1}{2}} = \left(\frac{a^2}{a^2 + \frac{a_M^2}{2^2}} \right)^{\frac{1}{2}} = f(a^2) \quad (7)$$

while $\mathbf{a}^2 = a^2$ represents scalar product between total acceleration vector \mathbf{a} with itself. It can be added that mentioned distinction between inertial and gravitational mass can be used for the "dark matter" effect modeling.

Secondly, we shall suppose the following form for system momentum \mathbf{p} and kinetic energy E_k

$$\mathbf{p} = m_i \mathbf{v} = m_g \mathbf{v} f(a^2) \quad (8)$$

$$E_k = \frac{m_i \mathbf{v}^2}{2} = \frac{m_g \mathbf{v}^2}{2} f(a^2) \quad (9)$$

where \mathbf{v} represents the system total velocity so that $\mathbf{a} = \frac{d\mathbf{v}}{dt}$. Obviously these kinematic variables (that do not include system potential energy and its coordinate derivations) can be simply obtained by introduction of (7) instead usual mass in the corresponding expression in usual Newtonian dynamics.

Thirdly, suppose that gravitational potential energy of the system in of the gravitational field of the point like source with mass M at distance r in respect to system is given by usual form

$$V_g = -\frac{Gm_g M}{r} \quad (10)$$

so that gravitational force

$$\mathbf{F} = -gradV_g = -\frac{Gm_g M}{r^2} \mathbf{r}_0 \quad (11)$$

has usual form of a central force, where $\mathbf{r} = r\mathbf{r}_0$ represents the coordinate vector of the system while \mathbf{r}_0 represent unit vector in \mathbf{r} direction.

Fourthly, suppose that Lagrangian of the system has the following form

$$L = E_k - V_g = \frac{m_g \mathbf{v}^2}{2} f(a^2) + \frac{Gm_g M}{r} \quad (12)$$

and that dynamical equations of the system can be obtained by variation calculus of this Lagrangian. These equations, since Lagrangian depends of a , are generalized Euler-Lagrange, i.e. Euler-Poisson equations

$$\frac{\partial L}{\partial x_q} - \frac{d}{dt} \frac{\partial L}{\partial v_q} + \frac{d^2}{dt^2} \frac{\partial L}{\partial a_q} = 0 \quad \text{for } q = 1, 2, 3 \quad (13)$$

or, according to (9), (10), (12),

$$\frac{\partial V_g}{\partial x_q} - \frac{d}{dt} \frac{\partial E_k}{\partial v_q} + \frac{d^2}{dt^2} \frac{\partial L}{\partial a_q} = 0 \quad \text{for } q = 1, 2, 3 \quad (14)$$

where x_q , v_q and a_q for $q = 1, 2, 3$ are corresponding components of the system radius vector, velocity and acceleration vector.

Obviously, suggested generalized dynamics of the system (6)-(14) is in general case different from usual Newton dynamics. This difference basically originates from (6) only, i.e. only from difference between inertial and gravitational mass. But effectively this difference manifests as a change and extension of the kinetic terms in the usual Newton dynamical equations. For this reason, as well as for reasons that will be discussed later, this generalized dynamics will be called k-MOND.

Consider limit of k-MOND in case of large accelerations absolute value, i.e. for

$$a^2 = a^2 \gg a_M^2 > \frac{a_M^2}{2^2}. \quad (15)$$

Then, as it is not hard to see, $f(1a^2)$ tends toward 1 and suggested generalized dynamics toward usual Newtonian dynamics in which, as well as in its relativistic generalizations, there is identity between gravitational and inertial mass, in full agreement with Eotvos experiments (realized definitely in large acceleration limit).

Consider, however, opposite limit of the k-MOND in case of small accelerations, i.e. for

$$a^2 = a^2 \ll \frac{a_M^2}{2^2}. \quad (16)$$

More precisely, consider an especial case of this limit in which system rotates uniformly. For reason of the formal simplicity, that does not any diminishing of the general conclusions, we shall firstly suppose that such uniform rotation exists and then we shall prove that dynamical equations (13), i.e. (14), are consistent with such supposition. Also, for reason of similar formal simplicity we shall suppose that system rotates in xOy plane where $x_1 = x$ and $x_2 = y$ while O denotes the coordinate beginning.

According to mentioned suppositions system radius vector \mathbf{r} has form

$$\mathbf{r} = r(\cos[\omega t]\mathbf{e}_1 + \sin[\omega t]\mathbf{e}_2 = r\mathbf{r}_0. \quad (17)$$

Here r and ω represent time independent orbit radius and angular frequency while \mathbf{e}_1 and \mathbf{e}_2 represent unit vectors in Ox_1 and Ox_2 directions while

$$\mathbf{r}_0 = \cos[\omega t]\mathbf{e}_1 + \sin[\omega t]\mathbf{e}_2 \quad (18)$$

represents time dependent unit vector in \mathbf{r} direction. Also, we can define time dependent perpendicular to \mathbf{r}_0 tangential unit vector

$$\mathbf{r}_t = -\sin[\omega t]\mathbf{e}_1 + \cos[\omega t]\mathbf{e}_2 \quad (19)$$

so that

$$\frac{d}{dt}\mathbf{r}_0 = \omega\mathbf{r}_t \quad (20)$$

and

$$\frac{d}{dt}\mathbf{r}_t = -\omega\mathbf{r}_0. \quad (21)$$

Then system velocity \mathbf{v} has form

$$\mathbf{v} = \frac{d}{dt}\mathbf{r}_t = \omega r \mathbf{r}_t = v \mathbf{r}_t \quad (22)$$

where velocity absolute value $v = \omega r$ is time independent. Of course, in any time moment, this velocity is perpendicular to radius vector so that $\mathbf{r}\mathbf{v} = 0$.

Further system acceleration vector has form

$$a = \frac{d}{dt}\mathbf{v} = -\omega^2 r \mathbf{r}_0 = -\frac{v^2}{r} \mathbf{r}_0 = -a \mathbf{r}_0 \quad (23)$$

with time independent absolute value $a = \frac{v^2}{r}$. Of course, in any time moment, this acceleration vector, parallel to radius vector, is perpendicular to velocity so that $a\mathbf{v} = \mathbf{0}$. Moreover, in any time moment, acceleration vector is perpendicular to its first time derivation so that $\mathbf{a}\frac{d\mathbf{a}}{dt} = 0$.

All this implies that system kinetic energy (9) in small acceleration limit can be approximated by the following expression

$$E_k = m_g \mathbf{v}^2 \frac{(a^2)^{\frac{1}{2}}}{a_M} = m_g \frac{\mathbf{v}^2}{a_M} (-a \mathbf{r}_0). \quad (24)$$

Then dynamical equations (14) turn out approximately in

$$\frac{\partial V_g}{\partial \mathbf{r}} - \frac{d}{dt} \frac{\partial E_k}{\partial \mathbf{v}} + \frac{d^2}{dt^2} \frac{\partial E_k}{\partial a} = 0 \quad (25)$$

or, as it is not hard to see, according to (17)-(23), in

$$\left(-\frac{Gm_g M}{r^2} \mathbf{r}_0\right) - \left(-2 \frac{m_g}{a_M} \frac{v^4}{r^2} \mathbf{r}_0\right) + \left(\frac{m_g}{a_M} v^4 r^2 \mathbf{r}_0\right) = 0 \quad (26)$$

and, finally, in

$$\left[-\frac{Gm_g M}{r^2} - \left(-\frac{m_g}{a_M} v^4 r^2\right)\right] \mathbf{r}_0 = 0. \quad (27)$$

It implies scalar equation

$$\frac{Gm_g M}{r^2} = \frac{m_g}{a_M} v^4 r^2 \quad (28)$$

identical to basic MOND proposition (2).

In this way it is definitely proved a possibility that Milgrom's modified Newtonian dynamics, MOND, can be consistently interpreted as a theory with the modified kinetic terms of the usual Newtonian dynamics (13), (14), simply called k-MOND.

In conclusion we can shortly repeat and point out the following. In this work we definitely prove a possibility that Milgrom's modified Newtonian dynamics, MOND, can be consistently interpreted as a theory with the modified kinetic terms of the usual Newtonian dynamics, simply called k-MOND. Precisely, we suggest only a functional dependence between inertial and gravitational mass tending toward identity in the limit of large accelerations (characteristic for Newtonian dynamics and its relativistic generalizations) but which behaves as a principal non-identity in the limit of small accelerations (smaller than Milgrom's acceleration constant). This

functional dependence implies a generalization of the kinetic terms (without any change of the gravitational potential energy terms) in the usual Newtonian dynamics including generalization of corresponding Lagrange formalism. Such generalized dynamics, k-MOND, is identical to Milgrom's MOND. Also, mentioned k-MOND distinction between inertial and gravitational mass would be formally treated as "dark matter".

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